Warm inflation with back - reaction: a stochastic approach

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I study a stochastic approach for warm inflation considering back - reaction of the metric with the fluctuations of matter field. This formalism takes into account the local inhomogeneities of the spacetime in a globally flat Friedmann - Robertson - Walker metric. The stochastic equations for the fluctuations of the matter field and metric are obtained. Finally, the dynamics for the amplitude of these fluctuations in a power - law expansion for the universe are examined.

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Recently, Berera and Fang [1] showed how thermal fluctuations may play the dominant role in producing the initial perturbations during inflation. They invoked slow - roll conditions. This ingenious idea was extended in some papers [2] into the warm inflation scenario. This scenario served as an explicit demonstration that inflation can occur in presence of a thermal component. However, the radiation energy density ρ_r must be small with respect to the matter energy density ρ_φ. More exactly, the kinetic component of the energy density (ρ_{kin}) must be small with respect to the vacuum energy density.

An alternative formalism for warm inflation was developed in previous works [3]. However, in these works is not taken into account the back - reaction of the metric with the fluctuations of the matter field. The aim of this letter is to include the back - reaction of the metric in the formalism previously developed in [3]. In the warm inflation scenario the kinetic energy density (ρ_{kin}) is small with respect to the vacuum energy, which is given by the density of potential energy $V(\varphi)$

$$\rho(\varphi) \sim \rho_m \sim V(\varphi) \gg \rho_{kin}.$$

where $\rho_{kin} = \rho_r(\varphi) + \frac{1}{2}\dot{\varphi}^2$, ρ_m is the matter energy density and

$$\rho_r(\varphi) = \frac{\tau(\varphi)}{8H(\varphi)}\dot{\varphi}^2, \tag{1}$$

is the radiation energy density. Here, $\tau(\varphi)$ and $H(\varphi)$ are the φ - dependent friction and Hubble parameters.

The density Lagrangian that describes the warm inflation scenario is

$$\mathcal{L}(\varphi, \varphi_{,\mu}) = -\sqrt{-g}\left[\frac{R}{16\pi} + \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + V(\varphi)\right] + \mathcal{L}_{int}. \tag{2}$$

Here, R is the scalar curvature, $g^{\mu\nu}$ is the metric tensor and g is the metric. The Lagrangian $\mathcal{L}_{int}$ takes into account the interaction of the field φ with other particles of the thermal bath. The interaction of φ with the thermal bath is represented as a friction parameter in eq. (1). All particlelike matter which existed before inflation would have been dispersed by inflation. If the thermal bath is sufficiently large, it will act as a heat reservoir which induces fluctuations on the inflaton field. In warm inflation the mean temperature of this bath (T_{ra}), must be smaller than the Grand Unified Theories (GUT) one: $T_{ra} < T_{GUT} \sim 10^{15}$ GeV. This condition implies that magnetic monopole suppression works effectively.

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As in previous works [4,5], I consider a semiclassical expansion for the scalar field $\varphi(\vec{x},t)$: $\varphi(\vec{x},t)=\phi_c(t)+\phi(\vec{x},t)$. Here, $< E|\varphi|E >= \phi_c(t)$ is the expectation value of the operator φ in an arbitrary state $|E>$. Furthermore, one requires that $< E|\phi(\vec{x},t)|E > = < E|\dot{\phi}(\vec{x},t)|E >= 0$. To consider the quantum fluctuations of the metric, one takes into account a quantum perturbed flat Friedmann - Robertson - Walker (FRW) metric $ds^2=-dt^2+A^2(\vec{x},t)d\vec{x}^2$ where $A(\vec{x},t)$ is a perturbed scale factor of the universe: $A(\vec{x},t)=a_o\, e^{\int H[\varphi(\vec{x},t)]\ dt}$.

Assuming a first order semiclassical expansion, the Hubble parameter becomes $H(\varphi)=H_c(\phi_c)+\left.\frac{dH(\varphi)}{d\varphi}\right|_{\phi_c}\phi\equiv H_c+H'\phi$. With this expansion for $H(\varphi)$ one obtains the expression for the quantum fluctuations of the metric $h(\vec{x},t)\simeq 2\int^t dt' H'\ \phi(\vec{x},t')$ (for small values of h), for the metric $ds^2=-dt^2+a^2(t)\left[1+h(\vec{x},t)\right]d\vec{x}^2$. Due to $< E|h(\vec{x},t)|E >= 0$, the expectation value for the metric $< E|ds^2|E >= -dt^2+a^2(t)d\vec{x}^2$ gives a globally flat FRW metric.

The quantum equation of motion for the operator φ is

$$\ddot{\varphi}-\frac{1}{< E|A|E >^2}\nabla^2\varphi+\left[3H(\varphi)+\tau(\varphi)\right]\ \dot{\varphi}+V'(\varphi)=0, \tag{3}$$

where $< E|A(\vec{x},t)|E >^2= a^2(t)$ for $A^2(\vec{x},t)\simeq a^2(t)\left[1+h(\vec{x},t)\right]$. The semiclassical Friedmann equation for a globally flat FRW metric is

$$\left\langle E|H^2(\varphi)|E\right\rangle=\frac{8\pi}{3M_p^2}\left\langle E|\rho_m(\varphi)+\rho_r(\varphi)|E\right\rangle, \tag{4}$$

where the matter and radiation energy densities are

$$\rho_m(\varphi)=\frac{\dot{\varphi}^2}{2}+\frac{1}{2a^2}\left(\vec{\nabla}\varphi\right)^2+V(\varphi), \tag{5}$$

$$\rho_r(\varphi)=\frac{\tau(\varphi)}{8H(\varphi)}\dot{\varphi}^2. \tag{6}$$

The φ dependent parameter $\tau(\varphi)$ describes the spatially inhomogeneous friction during the expansion. This friction becomes from the interaction of the matter field with the fields of the thermal bath. The following procedure consist in equating the eqs. (3) and (4) at same order in ϕ to obtain the dynamics of the system, once one makes the semiclassical expansions:

$$\varphi=\phi_c+\phi, \tag{7}$$

$$H(\varphi)=H_c(\phi_c)+H'\phi, \tag{8}$$

$$\tau(\varphi)=\tau_c(\phi_c)+\tau'\phi, \tag{9}$$

$$V(\varphi)=V(\phi_c)+V'\phi+1/2V''\phi^2, \tag{10}$$

$$V'(\varphi)=V'(\phi_c)+V''\phi+1/2V'''\phi^2. \tag{11}$$

We consider the eq. (3) at zero, first and second order in ϕ. These equations with $\tau(\varphi)=\gamma H(\varphi)$ [i.e., for $\tau_c(\phi_c)=\gamma H_c(\phi_c)$ and $\tau'(\phi_c)\phi=\gamma H'(\phi_c)\phi$] are

$$\ddot{\phi}_c+3H_c(\phi_c)[1+\gamma/3]\dot{\phi}_c+V'(\phi_c)=0, \tag{12}$$

$$\ddot{\phi}-\frac{1}{a^2}\nabla^2\phi+3H_c\left[1+\gamma/3\right]\dot{\phi}+\left[3H'\left(1+\gamma/3\right)\dot{\phi}_c+V''\right]\phi=0, \tag{13}$$

$$3H'\left(1+\gamma/3\right)\phi\dot{\phi}+V'''\phi^2=0, \tag{14}$$

where γ is a dimensionless constant which is a parameter of the theory. For $\gamma=0$ one recovers the standard inflation scenario where the bath is with zero temperature [5]. In this case the Lagrangian $\mathcal{L}_{int}$ in (2) can be neglected. The particular choice $\tau_c=\gamma H_c$ for the friction parameter, becomes from the requirement that the interaction of the matter field with the particles of the bath must decrease with time for the thermodynamic equilibrium holds at the end of inflation. Since $\dot{H}_c=-\frac{M_p^2}{4\pi}(H_c')^2\left(1+\frac{\tau_c}{3H_c}\right)^{-1}<0$, the thermodynamic equilibrium is garantized with the choice $\tau_c=\gamma H_c$. A more general study for the back - reaction of the metric with the matter field fluctuations will be developed in a further work [6].

From eq. (4), the Friedmann equations at zero, first and second order in ϕ, are

$$H_c^2(\phi_c) = \frac{4\pi}{3M_p^2}\left[\left(1+\frac{\gamma}{4}\right)\dot{\phi}_c^2 + 2V(\phi_c)\right], \tag{15}$$

$$\langle H_c H' \phi\rangle = \frac{4\pi}{3M_p^2}\left\langle \dot{\phi}_c\dot{\phi}\,(1+\gamma/4) + V'\phi\right\rangle = 0, \tag{16}$$

$$\frac{\mathcal{K}}{a^2} = \left\langle (H')^2\phi^2\right\rangle = \frac{4\pi}{3M_p^2}\left\langle \dot{\phi}^2\,(1+\gamma/4) + \frac{1}{a^2}\left(\vec{\nabla}\phi\right)^2 + V''\phi^2\right\rangle. \tag{17}$$

The eq. (17) gives the effective curvature ($\mathcal{K}$) of spacetime due to the back - reaction of the metric with the fluctuations of the matter field. Note that this curvature depends on the constant γ (i.e., depends on the friction parameter τ_c). The classical potential is

$$V(\phi_c) = \frac{3M_p^2}{8\pi}\left[H_c^2(\phi_c) - \frac{M_p^2}{12\pi}\left(H_c'\right)^2\left(1+\frac{\gamma}{4}\right)\left(1+\frac{\gamma}{3}\right)^{-2}\right]. \tag{18}$$

The eq. (13), with $\dot{\phi}_c = -\frac{M_p^2}{4\pi}H_c'\left(1+\frac{\gamma}{3}\right)^{-1}$ and (14), is

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + R(t)\dot{\phi} = 0, \tag{19}$$

where

$$R(t) = 3H_c\left(1+\frac{\gamma}{3}\right) + \frac{(1+\gamma/4)}{(1+\gamma/3)}\frac{M_p^2 H_c' V''}{\left(4\pi V' - 3M_p^2 H_c' H_c\right)} - \frac{3M_p^4 (H')^3}{\left(4\pi V' - 3M_p^2 H_c' H_c\right)}, \tag{20}$$

and $H_c' \equiv \frac{d}{d\phi_c}H_c(\phi_c)$. One can redefine the quantum fluctuations with the map $\chi = e^{1/2\int R(t)dt}\phi$, and the eq. (19) becomes

$$\ddot{\chi} - \frac{1}{a^2}\nabla^2\chi - \frac{k_o^2}{a^2}\chi = 0,$$

with

$$k_o^2(t) = a^2\left[\frac{1}{2}\left(\frac{R^2}{2} + \dot{R}(t)\right)\right]. \tag{21}$$

The redefined quantum fluctuations for the matter field and the fluctuations of the metric, written as a Fourier expansion, are

$$\chi(\vec{x},t) = \frac{1}{(2\pi)^{3/2}}\int d^3k\left[a_k\chi_k + a_k^{\dagger}\chi_k^*\right], \tag{22}$$

$$h(\vec{x},t) = \frac{1}{(2\pi)^{3/2}}\int d^3k\left[a_k h_k + a^{\dagger}h_k^*\right], \tag{23}$$

where $\chi_k(\vec{x},t) = \xi_k(t)e^{i\vec{k}.\vec{x}}$, $h_k(\vec{x},t) = \tilde{\xi}_k(t)e^{i\vec{k}.\vec{x}}$ and $\tilde{\xi}_k(t) = 2\int H'(t)\xi_k e^{-1/2\int^t R(t')dt'}\,dt$. Furthermore a_k and $a_k^{\dagger}$ are the annihilation and creation operators with commutation relations $\left[a_k, a_{k'}^{\dagger}\right] = \delta^{(3)}(k-k')$. The commutation relation between χ and $\dot{\chi}$ is $[\chi(\vec{x},t),\dot{\chi}(\vec{x}',t)] = \mathrm{i}\delta^{(3)}(\vec{x}-\vec{x}')$, and thus one obtains $\left[h(\vec{x},t),\dot{h}(\vec{x}',t)\right] = \frac{1}{(2\pi)^3}\int d^3k\left(\tilde{\xi}_k(t)\dot{\tilde{\xi}}_k(t) - \dot{\tilde{\xi}}_k(t)\tilde{\xi}_k(t)\right)e^{-i\vec{k}.(\vec{x}-\vec{x}')}$. The equation of motion for the time dependent modes is $\ddot{\xi}_k(t) + a^{-2}(t)\left[k^2 - k_o^2(t)\right]\xi_k(t) = 0$.

To study the matter and metric fluctuations we can separate the spectrum in both, the ultraviolet (UV) ($k > \epsilon k_o$) and the infrared (IR) ($k < \epsilon k_o$) sectors (where $\epsilon \ll 1$ is a dimensionless constant). In the IR sector both, $\chi(\vec{x},t)$ and $h(\vec{x},t)$ are classical, while in the UV sector they are quantized. Since $\ddot{\xi}_k - \frac{k_o^2}{a^2}\xi_k \simeq 0$, in the IR sector the classical stochastic equation that describes the dynamics of the fluctuations χ_{cg} is (for $\chi = \chi_{cg} + \chi_S$)

$$\ddot{\chi}_{cg} - \frac{k_o^2}{a^2}\chi_{cg} = \epsilon\left[\frac{d}{dt}\left(\dot{k}_o\eta\right) + 2\dot{k}_o\kappa\right], \tag{24}$$

where

$$\chi_{cg}(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\ \theta(\epsilon k_o - k) \left[a_k \chi_k + a_k^\dagger \chi_k^*\right], \tag{25}$$

$$\eta(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\ \delta(\epsilon k_o - k) \left[a_k \chi_k + a_k^\dagger \chi_k^*\right], \tag{26}$$

$$\kappa(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\ \delta(\epsilon k_o - k) \left[a_k \dot{\chi}_k + a_k^\dagger \dot{\chi}_k^*\right]. \tag{27}$$

Note that k_o in eq. (24) depends on the friction parameter $\tau_c = \gamma H_c$ [see also eqs. (21) and (20)]. When $(\dot{k}_o)^2 \left<(\kappa)^2\right> \ll (\ddot{k}_o)^2 \left<(\eta)^2\right>$ (i.e., for $\left|\frac{\dot{k}_o \dot{\xi}_{\epsilon k_o}}{\ddot{k}_o \xi_{\epsilon k_o}}\right| \ll 1$ [5]), we can neglect the noise κ with respect to the another noise (η) in (24) and the stochastic equation of motion for the coarse - grained field that describes the fluctuations of the metric in the IR sector is (for $h = h_{cg} + h_S$)

$$\ddot{h}_{cg} = \left[2\frac{d}{dt}(H') - H' R(t)\right] e^{-1/2\int R(t)dt} \chi_{cg} + 2H' e^{-1/2\int R(t)dt} \left[u + \epsilon \dot{k}_o \eta\right], \tag{28}$$

where $\dot{u} = \frac{k_o^2}{a^2}\chi_{cg}$ and

$$h_{cg}(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\ \theta(\epsilon k_o - k) \left[a_k h_k + a_k^\dagger h_k^*\right].$$

On the other hand, the quantum stochastic equation that determines the dynamics of the quantum fluctuations of the matter field χ_S in the UV sector [where it is valid the equation for the time dependent modes $\ddot{\xi}_k + a^{-2}[k^2 - k_o^2(t)]\xi_k = 0$ and the commutation relation is $[\chi_S, \dot{\chi}_S] = \mathrm{i}\delta^{(3)}(\vec{x} - \vec{x}')$], is

$$\ddot{\chi}_S - \frac{k_o^2}{a^2}\chi_S = -\epsilon \left[\frac{d}{dt}\left(\dot{k}_o \eta_q\right) + 2\dot{k}_o \kappa_q\right], \tag{29}$$

where

$$\chi_S(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\ \theta(k - \epsilon k_o) \left[a_k \chi_k + a_k^\dagger \chi_k^*\right], \tag{30}$$

$$\eta_q(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\ \delta(k - \epsilon k_o) \left[a_k \chi_k + a_k^\dagger \chi_k^*\right], \tag{31}$$

$$\kappa_q(\vec{x},t) = \frac{1}{(2\pi)^{3/2}} \int d^3k\ \delta(k - \epsilon k_o) \left[a_k \dot{\chi}_k + a_k^\dagger \dot{\chi}_k^*\right]. \tag{32}$$

Now we consider the special case of a power - law expansion of the universe for which $a \sim (t/t_o)^p$ and $H_c(t) = p/t$. The squared wavenumber that separates both, the UV and IR sectors, is

$$k_o^2(t) = H_o^{-2}\ t^{2(p-1)} K^2(\gamma,p), \tag{33}$$

where

$$K^2(\gamma,p) = 9/4p^2(1+\gamma/3)^2 - 3/2p(1+\gamma/3) + \frac{3p^2 M_p^2}{2\pi m^2}\left(1 - \frac{M_p^2}{48\pi^2 m^2}(1+\gamma/4)(1+\gamma/3)^{-2}\right). \tag{34}$$

The general solution for the time dependent modes $\xi_k(t)$ is

$$\xi_k(t) = A_1\sqrt{t/t_o}\ H_\nu^{(1)}\left[\frac{H_o k\ (t/t_o)^{1-p}}{p-1}\right] + A_2\sqrt{t/t_o}\ H_\nu^{(2)}\left[\frac{H_o k\ (t/t_o)^{1-p}}{p-1}\right], \tag{35}$$

where $H_\nu^{(1)}$ and $H_\nu^{(2)}$ are the first and second species Hankel functions and $\nu = \frac{1}{2(p-1)}\sqrt{1+4K^2(\gamma,p)}$. We choose $A_1 = 0$. For $p > 1$ (i.e., for $\nu > 3/2$, $\gamma \ll 1$ and $t \gg 1$), one obtains the asymptotic solution

$$\xi_k(t) = A_2\sqrt{t/t_o}\left[\frac{1}{\sqrt{2\pi\nu}} e^{\nu}\left(\frac{H_o k\ (t/t_o)^{1-p}}{2\nu(p-1)}\right)^{\nu} + \mathrm{i}\sqrt{\frac{2}{\pi\nu}} e^{-\nu}\left(\frac{H_o k\ (t/t_o)^{1-p}}{2\nu(p-1)}\right)^{-\nu}\right], \tag{36}$$

where e denotes the exponential number. From the commutation condition $[\chi(\vec{x},t),\dot{\chi}(\vec{x}',t)] = \frac{1}{(2\pi)^3}\int d^3k\ (\xi_k\dot{\xi}_k^* - \dot{\xi}_k\xi_k^*)\ e^{-i\vec{k}.(\vec{x}-\vec{x}')} = \mathrm{i}\delta^{(3)}(\vec{x}-\vec{x}')$ one obtains the value $A_2 = \frac{\mathrm{i}}{2}\sqrt{\frac{\pi}{p-1}}$.

In the IR sector ($k \ll \frac{2\nu(p-1)(t/t_o)^{p-1}}{H_o}$), the asymptotic time dependent modes become

$$\xi_k|_{IR} \simeq -\sqrt{t/t_o}\frac{e^{-\nu}}{2}\sqrt{\frac{2}{\nu(p-1)}}\left(\frac{H_o k\ (t/t_o)^{1-p}}{2\nu(p-1)}\right)^{-\nu}, \tag{37}$$

which are asymptotically real. On the other hand, in the UV sector the modes are

$$\xi_k|_{UV} \simeq \sqrt{t/t_o}\left[\mathrm{i}\frac{e^{\nu}}{2}\sqrt{\frac{1}{2\nu(p-1)}}\left(\frac{H_o k(t/t_o)^{1-p}}{2\nu(p-1)}\right)^{\nu} - \frac{e^{-\nu}}{2}\sqrt{\frac{2}{\nu(p-1)}}\left(\frac{H_o k(t/t_o)^{1-p}}{2\nu(p-1)}\right)^{-\nu}\right], \tag{38}$$

which are complex functions. The function that defines the transformation $\chi = e^{1/2\int R(t)dt}\phi$ is

$$R(t) = p\ t^{-1}\left[3(1+\gamma/3) + \frac{(1+\gamma/4)}{(1+\gamma/3)}\frac{M_p^2 m^{-2} C}{8\pi C - 3M_p^2} - \frac{3M_p^4 m^{-2}}{32\pi^2 C - 12\pi M_p^2}\right], \tag{39}$$

where $C = \frac{3M_p^2}{8\pi}\left[1 - \frac{M_p^2 m^{-2}}{12\pi}(1+\gamma/4)(1+\gamma/3)^{-2}\right]$.

The squared amplitude for the fluctuations of the metric in the IR sector go as

$$< h_{cg}^2 >\big|_{IR} \sim t^{\nu(p-1)-2(1+M)+3p},$$

with $M = 2p\left[3(1+\gamma/3) + \frac{(1+\gamma/4)}{(1+\gamma/3)}\frac{M_p^2 m^{-2} C}{8\pi C - 3M_p^2} - \frac{3M_p^4 m^{-2}}{32\pi^2 C - 12\pi M_p^2}\right]$. Note for $M > \nu/2(p-1) + 3/2p - 1$ the amplitude for the fluctuations of the metric in the IR sector decreases with time, but for $M < \nu/2(p-1) + 3/2p - 1$ these fluctuations increases. The squared amplitude for the fluctuations of the matter field go as $< \phi_{cg}^2 > \sim < h_{cg}^2 >$.

At the end of inflation (for $\phi_c = 0$) one obtains $H_c = H_o$ and the radiation temperature is

$$\frac{T_{ra}}{M_p} \simeq \left(\frac{15}{64\pi^3}\right)^{1/4}\left[\frac{\gamma H_o}{m\ N(T_{ra})(1+\gamma/3)}\right]^{1/4}, \tag{40}$$

where $N(T_{ra})$ is the number of relativistic degrees of freedom at temperature T_{ra} (at the end of inflation). For $N(T_{ra}) = 10^3$ and $H_o \simeq m$ one obtains the cutoff $\frac{T_{re}}{M_p} < 10^{-5}$ for $\gamma < 10^{-15}$. In this framework the constant γ can be interpreted as a effective coupling constant due to the interaction between the inflaton and the particles of the thermal bath.

Finiteness in the UV sector is achieved by imposing a cutoff at the horizon scale ($k \sim p/t$). Here, the amplitude for the fluctuations of the matter field decreases for $M > \nu(p-2) + 2$ and $\nu > 3/2$, due to $\left\langle h_S^2 \right\rangle \sim t^{2\nu(p-2)-2M+4}$.

In this letter I have studied a stochastic approach to the Warm Inflation scenario considering back - reaction of the metric due to the fluctuations of the matter field $\phi(\vec{x},t)$. In this theory the classical matter field $< E|\varphi|E >= \phi_c$ lead to the expansion of the universe, while the fluctuations of the matter field generate an effective curvature ($\mathcal{K}$), on the background flat FRW metric [see eq. (17)]. However, global curvature of the spacetime is zero, due to $< E|h(\vec{x},t)|E >= 0$. In this framework the early universe is understood as expanding regions of the universe with locally different rate of expansion $H(\varphi) \simeq H_c(\phi_c) + H'(\phi_c)\phi(\vec{x},t)$, but with expectation value $< E|H(\varphi)|E >= H_c(\phi_c)$.

To summarize, second order stochastic equations for the fluctuations of the matter field in both, the IR and UV sectors, were obtained. Also, a second order stochastic equation for the fluctuations of the metric in the IR sector was founded. Furthermore, a new second order quantum stochastic equation for the matter field fluctuations in the UV sector was founded [see eq. (29)]. The evolution for the fluctuations of the matter field depends on the evolution of the superhorizon and the scale factor, which depends on the friction parameter that describes the interaction of the matter field with the fields in the thermal bath. I find that the amplitudes of both, the matter and metric fluctuations decreases with time in the IR sector (for $p > 1$ and $\gamma \ll 1$). This implies that the effects on the now observational scales should be a soft disturbed flat FRW spacetime. Such fluctuations should be explained by the quadrupole anisotropy amplitude in the cosmic background radiation spectrum. However, this topic is beyond the scope of this letter.

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